MJC Physics – Paper V (Mathematical Physics)

Semester IV | Unit–I (Coordinate Systems)

Topic: Cylindrical and Spherical Coordinate Systems

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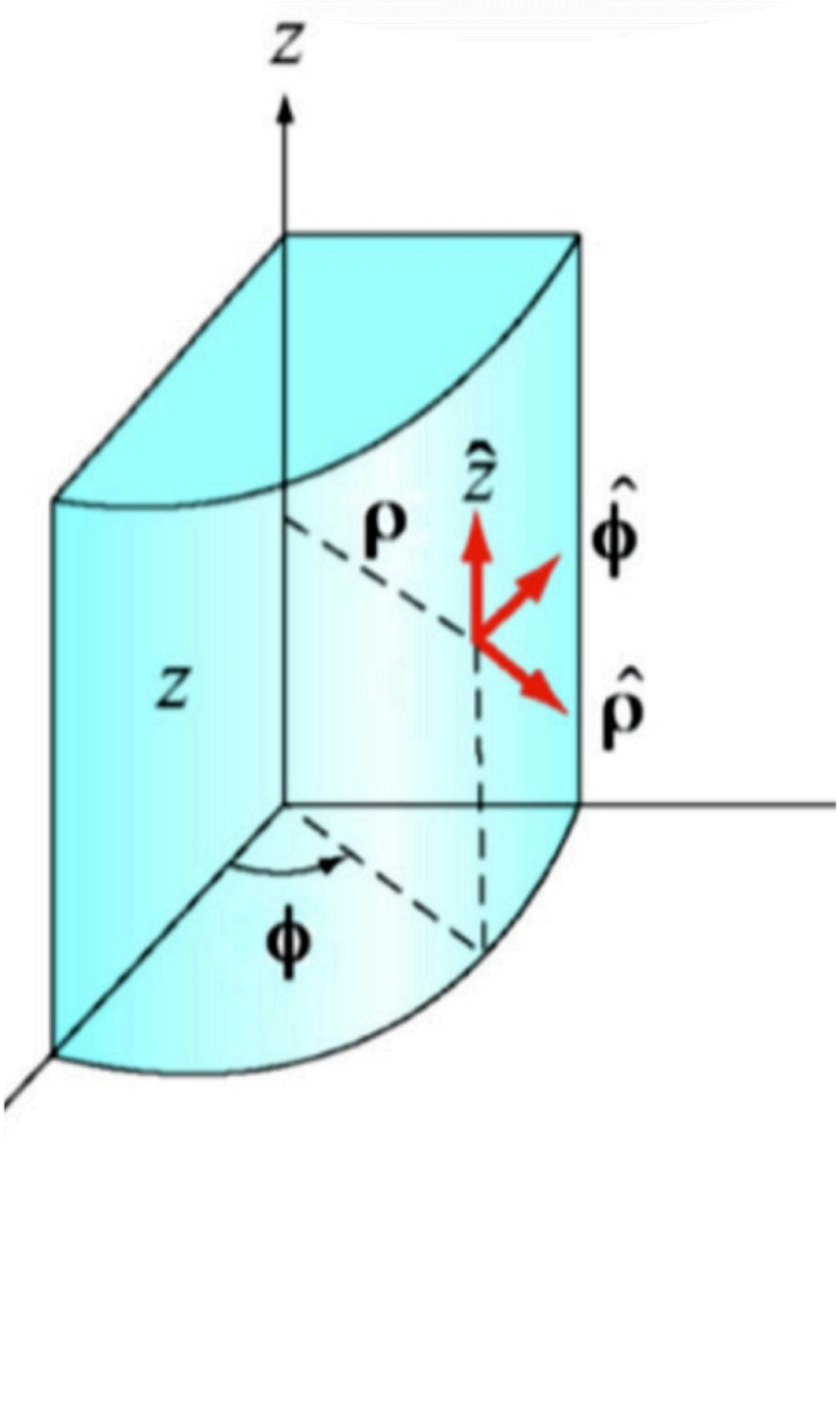
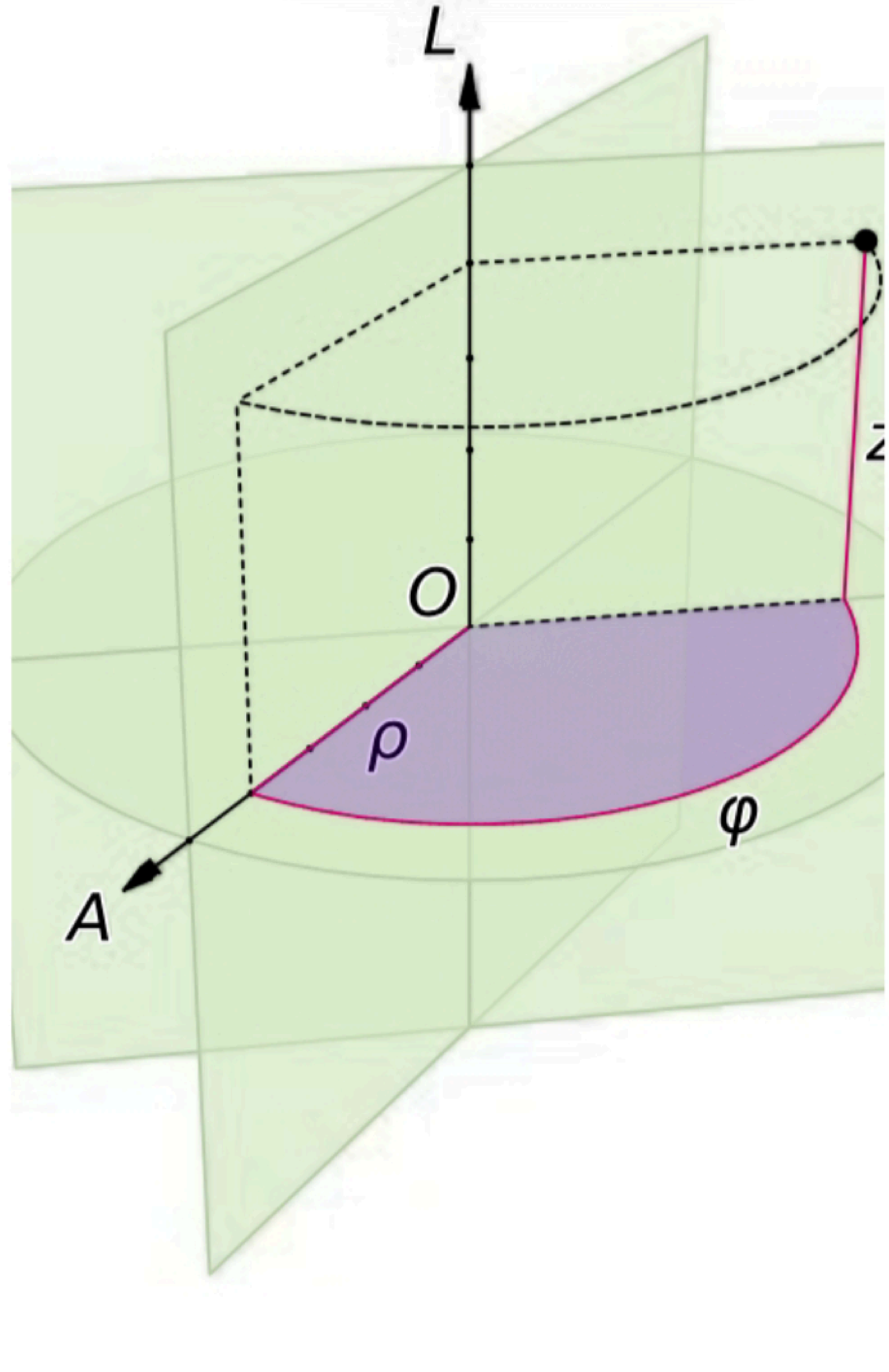
1. Introduction

In many physical problems, especially those involving symmetry, **Cartesian coordinates** are not the most convenient. Depending on the geometry of the system, **cylindrical** and **spherical coordinate systems** provide simpler mathematical descriptions. These systems are widely used in **electrostatics, gravitation, quantum mechanics, and fluid dynamics**.

2. Cylindrical Coordinate System $(r,;\phi,;z)$





2.1 Definition

In the cylindrical coordinate system, the position of a point is specified by:

- r: perpendicular distance from the z-axis
- : azimuthal angle measured from the positive x-axis
- z: height along the z-axis

This system is obtained by extending **plane polar coordinates** along the z-direction.

2.2 Transformation Equations

Cylindrical to Cartesian

$$x = r\cos\phi, \quad y = r\sin\phi, \quad z = z$$

Cartesian to Cylindrical

2.3 Differential Length, Area and Volume Elements

- **Line element**

- **Area element**

- **Volume element**

$$dV = r\,dr\,d\phi\,dz$$

2.4 Surfaces of Constant Coordinates

- $r = \text{constant}$: circular cylinder
- $\phi = \text{constant}$: vertical plane
- $z = \text{constant}$: horizontal plane

2.5 Applications

- Electric field due to an **infinite line charge**
- Magnetic field around a **long straight current-carrying conductor**
- Heat conduction in cylindrical rods

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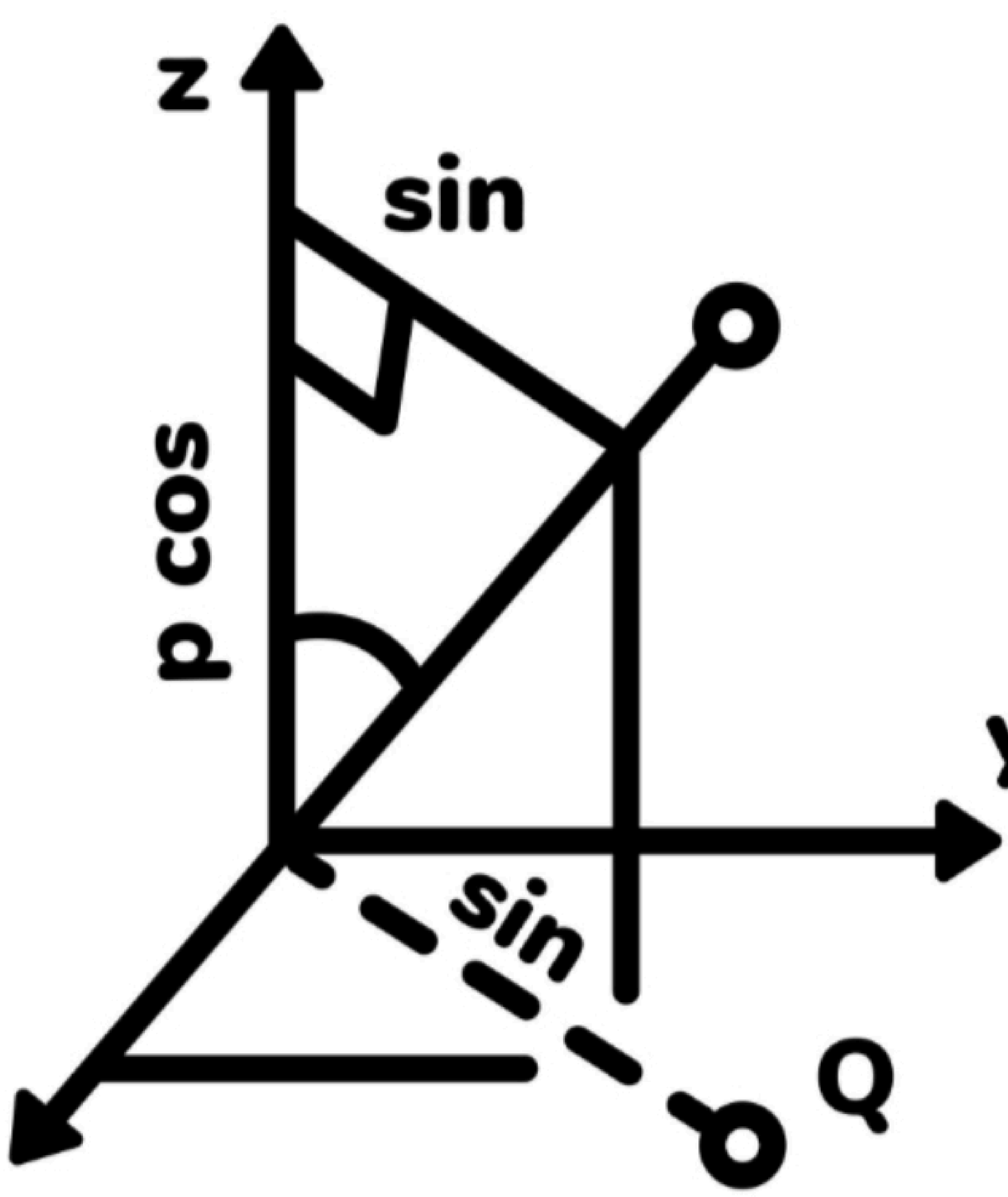
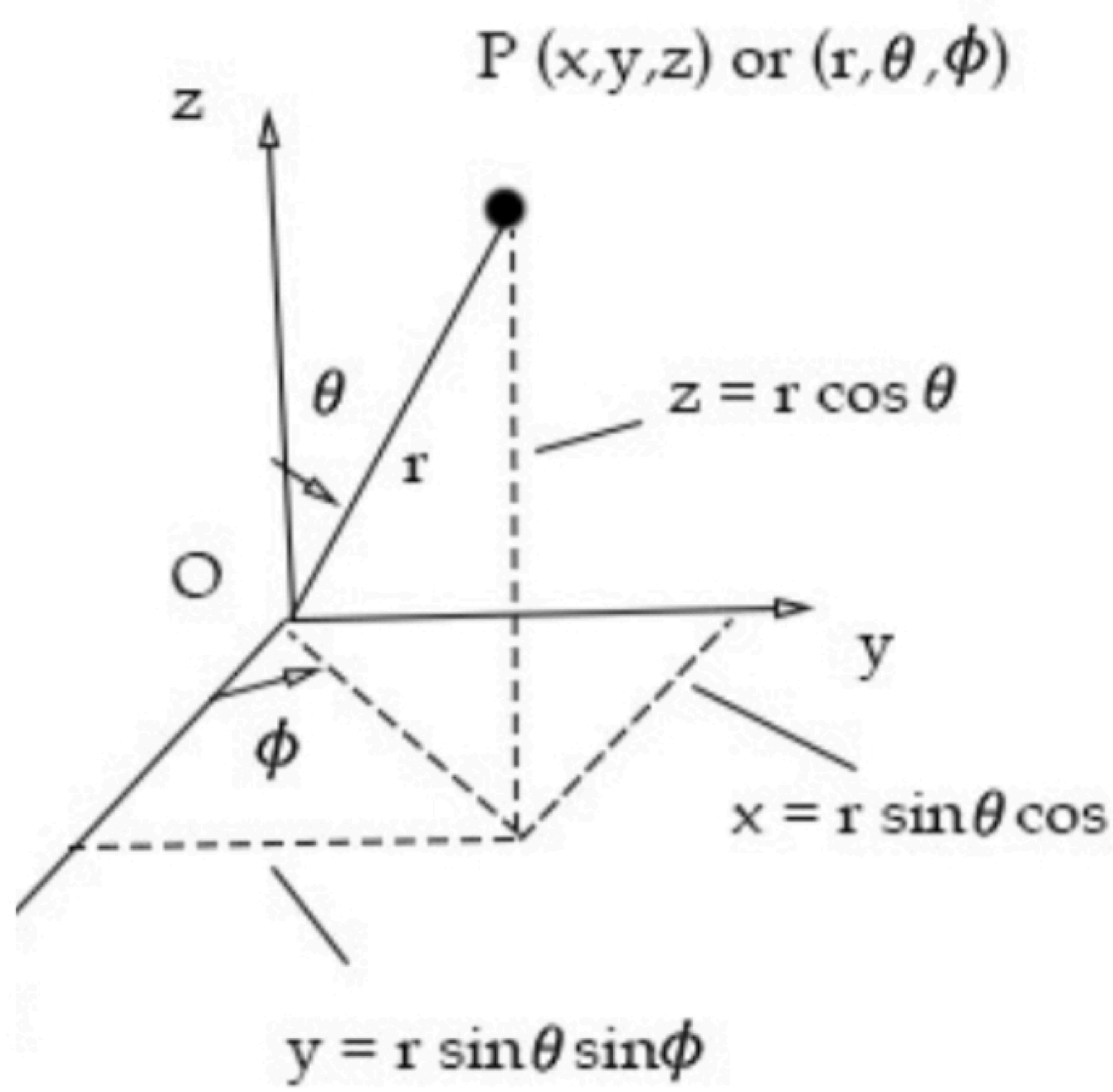
3. Spherical Coordinate System

$(r,;\theta,;\phi)$





herical Coordinat

3.1 Definition

In spherical coordinates, the position of a point is described by:

- r: distance from the origin
- θ: polar angle with the positive -axis
- : azimuthal angle in the -plane

This system is most suitable for problems having **spherical symmetry**.

3.2 Transformation Equations

Spherical to Cartesian

$$x = r\sin\theta\cos\phi$$

$$y = r\sin\theta\sin\phi$$

$$z = r\cos\theta$$

Cartesian to Spherical

$$\theta = \cos^{-1}\left(\frac{z}{r}\right)$$

3.3 Differential Length, Area and Volume Elements

- Line element

- Area element

- Volume element

3.4 Surfaces of Constant Coordinates

- : sphere
- $\theta = \text{constant}$: cone
- $\phi = \text{constant}$: vertical plane

3.5 Applications

- Electric field of a **point charge**
- Gravitational field of spherical bodies
- Hydrogen atom in **quantum mechanics**

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4. Conclusion

Cylindrical and spherical coordinate systems simplify mathematical treatment of physical problems with axial and spherical symmetry respectively. Mastery of coordinate transformations and differential elements is essential for advanced studies in physics.